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On $N = 4$ supersymmetric Yang–Mills in harmonic superspace

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Abstract. We present an analysis of $N=4$ supersymmetric Yang-Mills theory using a construction involving additional bosonic variables in the coset space SU(4)/H. No choice of H can be shown to lead to an analytic formulation of the theory. By introducing an analysis on dual planes we reduce the theory (including the reality constraint) to one involving $N=2$ symmetry. This approach has to be extended to include truly harmonic derivatives. For the typical case of SU(4)/SU(2) × U(1) prepotentials are introduced which solve the constraints. It has not been possible, however, to construct an action which leads to the equation of motion for the original $N=4$ supersymmetric Yang-Mills theory (at the linearised level).

A new method has been presented, that of harmonic superspace [1], which may allow the construction of full superspace versions of N-extended supersymmetric Yang-Mills and supergravity theories beyond the $N=3$ barrier [2]. This barrier arises due to counting arguments and shows that it is impossible to construct $N \geqslant 3$ extended supersymmetric Yang-Mills or supergravity theories using representations of the corresponding global supersymmetry algebra. The harmonic superspace method utilises the adjunction of extra, dimensionless variables belonging to the coset space SU(N)/H, where H is a suitably chosen subgroup of the internal symmetry group SU(N) of the N-extended supersymmetry algebra. For $N=2$, H was chosen to be U(1) [1]. Such a formulation has led to an elegant construction of $N=3$ supersymmetric Yang-Mills theory [3], using extra variables belonging to the coset space $SU(3)/[U(1)]^2$.

There is also another method available to broach the $N=3$ barrier, that of central charges [4]. This method involves the use of additional bosonic variables which now have the same dimensions as those of the spacetime coordinates. This method has allowed the construction of a full superspace version of $N=4$ supersymmetric Yang-Mills theory [5] and of $N=8$ supergravity [6]. However, the corresponding actions involve a degeneracy constraint which requires careful treatment of the quantisation of these theories. Whilst such a treatment now appears possible [7] it still appears of value to explore all other alternative methods of constructing super-theories beyond the $N=3$ barrier. In particular the harmonic superspace approach has the advantage for $N=2$ [1] and $N=3$ [3] of using unconstrained superfields. We would therefore expect the quantisation rules to be much simpler than in the central charge case [7].

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We might also hope thereby to be able to understand the finiteness results [8] in a more direct manner. We propose to analyse the extension of harmonic superspace to attempt the construction of an unconstrained superspace version of $N=4$ supersymmetric Yang–Mills (SYM) in this paper.

The superspace approach to constructing N-extended SYM theories is by means of the use of gauged superspace in which there are constraints on the field strengths F_{AB} (A runs over the values a (spacetime), $\boldsymbol{\alpha} = \overset{i}{\alpha}$, $\dot{\boldsymbol{\alpha}} = \dot{\alpha} i$, where we use 2-spinor notation and $1 \leqslant i \leqslant N$). The constraints for $N=4$ SYM, which give rise to the field equations [9], are

$$F^{(ij)}_{\alpha\beta} = F_{\dot{\alpha}(i\dot{\beta}j)} = F_{\boldsymbol{\alpha}\dot{\boldsymbol{\beta}}}{\sim} = 0 \tag{1}$$

where $\sim$ denotes traceless in the internal SU(4) indices. There is also the reality constraint, absent from the $N=3$ SYM case,

$$F^{\alpha i j}{}_{\alpha} = \tfrac{1}{2} F^{\dot{\alpha}}{}_{k\dot{\alpha} l}\, \varepsilon^{ijkl}. \tag{2}$$

It is this reality constraint which will cause new features to occur in our analysis, as already hinted at in [3]. We will come to these shortly.

The main thrust of the harmonic superspace approach is the idea of 'analytic' superfields. A general superfield $\Phi(X^a, \theta_{\boldsymbol{\alpha}}, \bar{\theta}_{\dot{\boldsymbol{\alpha}}})$ depends on all of its variables; an analytic one depends only on a subset of the anticommuting variables $\theta_{\boldsymbol{\alpha}}$, $\bar{\theta}_{\dot{\boldsymbol{\alpha}}}$. Thus the notion is an extension of that of a chiral superfield for $N=1$ superfields to the case of higher N. The explicit SU(N) internal symmetry may be preserved if the excluded θ or $\bar{\theta}$ variables are chosen as linear combinations of the ($\theta_{\boldsymbol{\alpha}}$, $\bar{\theta}_{\dot{\boldsymbol{\alpha}}}$) by use of the additional coset space variables u_i^L belonging to SU(N)/H. Here $1 \leqslant i \leqslant N$ as usual, and L runs over a set of N representations of the chosen subgroup H of SU(N). Thus for $N=2$ [1], H = U(1), $L = +1, -1$ (where $+1$ denotes the U(1) charge) and for $N=3$ [3], H = U(1) × U(1) and $L = ((1,1), (-1,1), (0,-2))$. For any H we may determine L by the decomposition of the fundamental representation N of SU(N) under H. Then certain of the linear combinations $\theta_\alpha^{-L} = \theta^i_\alpha u_i^{-L}$, $\bar{\theta}^L_{\dot{\alpha}} = \bar{\theta}_{\dot{\alpha} i} u_i^L$ may be chosen as the missing Grassmann variables in an analytic superfield, but the full internal SU(N) symmetry is not lost but is carried by the coset space variables u_i^L.

There are numerous possibilities for the choice of H for SU(4). Those constructed from unitary subgroups are listed in table 1, with the associated representations L. We have excluded cases where the same L must occur more than once, since the resulting matrix u_i^L will then be singular.

Table 1. The possible unitary subgroups H of SU(4) available for the construction of $N=4$ harmonic superspace, and the associated representations L into which the fundamental irrep **4** of SU(4) decomposes; $\boldsymbol{r}$ denotes the r-dimensional irrep of SU(2), SU(3) or USp(4).

H	L
$[\mathrm{U}(1)]^2$	$(1,1), (-1,1), (0,-2), (0,0)$
	$(1,1), (-1,1), (0,1), (0,-3)$
$[\mathrm{U}(1)]^3$	$(1,1,1), -1,1,1), (0,-2,1), (0,0,-3)$
SU(2) × U(1)	$(+1,\mathbf{2}), (-1,\mathbf{2})$
SU(2) × SU(2)	$(\mathbf{1},2), (\mathbf{2},1)$
SU(4)/U(3) = CP_3	$(-1,\mathbf{3}), (3,\mathbf{1})$
USp(4)	$(\mathbf{4})$

For a given H and choice of the set L we may introduce the associated spinorial derivatives $\mathscr{D}_\alpha^{+L} = \mathscr{D}^i_\alpha u_i^L$, $\bar{\mathscr{D}}_{\dot\alpha}^{-L} = \bar{\mathscr{D}}_{\dot\alpha i} u_i^L$, and the harmonic derivatives D^N, where in general $\mathscr{D}_\alpha = D_\alpha + igA_\alpha$, $\bar{\mathscr{D}}_{\dot\alpha} + igA_{\dot\alpha}$ in terms of the supersymmetric Yang-Mills coupling constant g and the spinorial components of the supersymmetric Yang-Mills gauge potentials A_α, $A_{\dot\alpha}$. The label N goes over the pairs of irreps L in the Abelian cases, as in the $N=2$ and 3 cases, but is closely related to, but not identical to those pairs, in the non-Abelian cases. Thus for $\mathrm{H}=[\mathrm{U}(1)]^3$ we have $N = L - M$, for pairs $L \neq M$, with

$$D^{L-M} = \bar{u}^{-Mi}\partial/\partial \bar{u}^{-Li} - u_i^L \partial/\partial u_i^M$$

and

$$[D^{L_1-M_1}, D^{L_2-M_2}] = D^{L_1+L_2-(M_1+M_2)} \tag{3}$$

if $L_1 + L_2 - (M_1 + M_2)$ is in the set of $L - M$, and is zero otherwise.

One of the purposes of introducing the generators $\mathscr{D}_\alpha^L$, $\bar{\mathscr{D}}_{\dot\alpha}^{-L}$, D^N is so as to rewrite the constraints (1) and (2) in a form which reduces them to lower N (less than 3), and hence allows an unconstrained superfield description to be given. Thus for $N=3$ [3] the constraints in harmonic form, for a suitably chosen subset of generators, reduce to those for $N=1$ SYM. The choice of the subset of spinorial generators to be analysed is determined by the requirement of independence of a suitably modified subset of variables formed from $x^{\alpha\dot\alpha}$ (rewriting the vector x^a in 2-spinor notation) and the θ, $\bar\theta$ variables. Thus we define

$$x_A^{\alpha\dot\alpha} = x^{\alpha\alpha} + 2\mathrm{i} \sum_L \varepsilon_L \theta^{-L\alpha} \bar\theta^{L\dot\alpha} \tag{4}$$

and use the SUSY transformation laws:

$$\delta x_A^{\dot\alpha\alpha} = 2\mathrm{i}\varepsilon_i^\alpha \sum_L u_i^{-L} \bar\theta^{L\dot\alpha}(\varepsilon_L - 1) - 2\mathrm{i}\varepsilon^{\dot\alpha i} \sum_L u_i^L \theta^{-L\alpha}(1+\varepsilon_L). \tag{5}$$

We can therefore exclude the occurrence of certain of the $\bar\theta^{L\dot\alpha}$, $\theta^{-L\alpha}$ entering $\delta x_A^{\alpha\dot\alpha}$ in (5) by a judicious choice of $\varepsilon_L = \pm 1$ as was achieved for $N=2$ [1] and $N=3$ [3]. We can see from (5) that we can remove a maximum of half of the θ, $\bar\theta$ variables. Thus for $N=4$ we reduce to eight spinorial degrees of freedom, and the measure $\mathrm{d}^4 x_A\, \mathrm{d}^8\theta$ will be dimensionless. We note that for $N>4$ the measure will always have positive (mass) dimension, so that the technique being used will require the use of prepotentials in the construction of actions.

Let us suppose we have reduced the set of variables on which we want our theory to depend to be θ^{-L_1}, θ^{-L_2}, $\bar\theta^{L_3}$, $\bar\theta^{L_4}$ in the case of Abelian H or H containing at most SU(2); if H contains an SU(3), as in the case of $\mathrm{H}=\mathrm{U}(3)$, then we must take θ^{-L_1}, θ^{-L_2}, θ^{-L_3}, $\bar\theta^{L_4}$ or similar. In order to implement analyticity it seems necessary to be able to construct a 'λ-τ bridge' [1, 3] which means to transform from the case when $\mathscr{D}_\alpha^L$, $\bar{\mathscr{D}}_{\dot\alpha}^{-L}$ contain the Yang-Mills potential by a pure gauge transformation to when they are independent of the potential. We thus require

$$[\mathscr{D}_{\alpha'}^{L_r}, \mathscr{D}_{\beta'}^{L_s}]_+ = 0 \tag{6}$$

for at least two values r, $s = 1, 2$ (or r, $s=3$ in addition for $\mathrm{H}=\mathrm{U}(3)$). But (6) requires that $F_{\alpha\beta}^{ij} = 0$, and not just (1), so that the SYM field strength superfield $F^{\alpha i j}{}_\alpha$ has been eliminated; the theory has become trivial. We must therefore require either that:

(a) (6) is only true for $r=s=1$ (say), and we can only consider explicitly the spinorial generators $\mathscr{D}_\alpha^{L_1}$, $\bar{\mathscr{D}}_{\dot\alpha}^{-L_3}$; or

(b) (6) is not true but we consider the generators $\mathscr{D}_{\alpha}^{L_1}$, $\mathscr{D}_{\alpha}^{L_2}$, $\bar{\mathscr{D}}_{\dot{\alpha}}^{-L_3}$, $\bar{\mathscr{D}}_{\dot{\alpha}}^{-L_4}$.

In the case (a) we do have only the constraints (1) but cannot implement the reality constraint (2) because we cannot obtain $F^{\alpha[ij]}_{\dot{\alpha}}$. We must therefore turn to the case (b), for which we cannot use the analytic framework associated with the 'λ-τ bridges' of the $N=2$ [1] and $N=3$ [3] constructions. In other words, our constraints cannot be reduced to those of $N=1$ SYM, and so will not be soluble in terms of a dimensionless prepotential superfield V but will require at least a dimensional prepotential like that for $N=2$ SYM [10]. However this need not prevent us writing down an action using such prepotentials together with suitable spinorial derivatives to have the correct dimensions.

We may proceed by analysing each choice of H of table 1 to see if it leads to a prepotential which is suitable for the construction of such an action leading to the constraints (1) and (2). Before we do this we will give an analysis of the constraints in terms of a more general construction than the use of harmonic coordinates. This approach will allow us to deduce a number of general statements and also may lead to an alternative analysis.

We proceed by introducing the notion of 'dual u-p planes', which is a translation into the internal symmetry space, appropriate for the reality constraint (2), of the notion of light-like lines introduced earlier by Witten [11]. This will give a minimal reformulation of the constraints (1) and (2). We use two sets of linearly independent variables $u_i^{(1)}$, $u_i^{(2)}$ and $p^{i(1)}$, $p^{i(2)}$ $(1\leqslant i\leqslant 4)$ with the duality condition

$$\sum^{1} \varepsilon^{ijkl} u_k^{(1)} u_l^{(2)} = p^{[i(1)}p^{j](2)}. \tag{7}$$

If we multiply (7) by $u_i^{(1)}$ we obtain $(\boldsymbol{u}^{(1)}\cdot\boldsymbol{p}^{(1)})\boldsymbol{p}^{(2)}=(\boldsymbol{u}^{(1)}\cdot\boldsymbol{p}^{(2)})\boldsymbol{p}^{(1)}$; since $\boldsymbol{p}^{(1)}$ and $\boldsymbol{p}^{(2)}$ are assumed linearly independent then they are each orthogonal to $\boldsymbol{u}^{(1)}$ (and similarly to $\boldsymbol{u}^{(2)}$; we are denoting by $\boldsymbol{u}^{(l)}$ the four-component vector $u_i^{(l)}$). We may define

$$\mathscr{D}_{\alpha}^{l} = \mathscr{D}^{i}_{\alpha} u_i^{(l)} \qquad \bar{\mathscr{D}}_{\dot{\alpha}m} = \bar{\mathscr{D}}_{\dot{\alpha}i} p^{i(m)} \tag{8}$$

and can rewrite (1) and (2) as

$$[\mathscr{D}^{l}_{\alpha}, \bar{\mathscr{D}}_{\dot{\beta}m}]_{+} = 0 \tag{9a}$$

$$[\mathscr{D}^{l}_{\alpha}, \mathscr{D}^{m}_{\beta}]_{+} = \varepsilon_{\alpha\beta}\varepsilon^{lm} W \tag{9b}$$

$$[\bar{\mathscr{D}}_{\dot{\alpha}l}, \bar{\mathscr{D}}_{\dot{\beta}m}]_{+} = \varepsilon_{\dot{\alpha}\dot{\beta}}\varepsilon_{lm} W \tag{9c}$$

for a 'real' superfield W under the conjugation $\theta_{\alpha}^{+l} \overset{*}{\leftrightarrow} \bar{\theta}_{\dot{\alpha}}^{+l}$. We may also introduce the generators $D_u^{\pm}$, D_u^{3}, $D_p^{\mp}$, D_p^{3} which conserve (7) as

$$D_u^{\pm} = u_i^{(1)}\partial/\partial u_i^{(2)} \pm u_i^{(2)}\partial/\partial u_i^{(1)} \qquad D_u^{3} = u_i^{(1)}\partial/\partial u_i^{(1)} - u_i^{(2)}\partial/\partial u_i^{(2)} \tag{10a}$$

$$D_p^{\mp} = p^{i(1)}\partial/\partial p^{i(2)} \pm p^{i(2)}\partial/\partial p^{i(1)} \qquad D_p^{3} = p^{i(1)}\partial/\partial p^{i(1)} - p^{i(2)}\partial/\partial p^{i(2)} \tag{10b}$$

where (D^{+}, D^{-} and D^{3}) correspond to $(\tau_1, i\tau_2, \tau_3)$. Let us relabel D^{+} and D^{-} as D^{1}, iD^{2}, so as to have SU(2) generators. Then besides the normal SU(2) commutation relations for (D_u^{a}), (D_p^{a}), with $1\leqslant a\leqslant 3$, we also have the commutators

$$[D_u^{a}, \mathscr{D}^{l}_{\alpha}]_{-} = (\tau^{a})^{lm}\mathscr{D}^{m}_{\alpha}, \qquad [D_p^{a}, \mathscr{D}^{l}_{\alpha}]_{-} = 0 \tag{11a}$$

$$[D_u^{a}, \bar{\mathscr{D}}_{\dot{\alpha}l}]_{-} = 0 \qquad [D_p^{a}, \bar{\mathscr{D}}_{\dot{\alpha}l}]_{-} = (\tau^{a})^{lm}\bar{\mathscr{D}}_{\dot{\alpha}m}. \tag{11b}$$

We have thus reformulated the set of constraints (1) and (2) with the internal SU(4) symmetry in terms of those with an internal SU(2)×SU(2) symmetry.

We may relate our present discussion to the coset space formulation if we take $u_i^{(1)}=u_i^{L_1}$, $u_i^{(2)}=u_i^{L_2}$, $p^{i(1)}=u_i^{L_3}$, $p^{i(2)}=u_i^{L_4}$ in the above u-p plane formulation. Thus we only appear able to obtain a satisfactory coset space construction for $N=4$ SYM in the case of H being at most the product $SU(2)\times SU(2)$; this excludes the case $SU(4)/U(3)=CP_3$. We note that $SU(4)/SU(2)\times SU(2)$ is the complex Grassmann manifold usually denoted $\Gamma^c_{4,2}$, being the collection of two-dimensional hyperplanes in C^4.

In order to proceed further we must decide which of the constraints (9), (11) and the SU(2) commutation relation for the D^a_u, D^a_p we wish to solve and which we wish to obtain from an action. To make a choice it seems most convenient to proceed in a similar manner to the harmonic method [1], [3], and treat all but (11) as constraints to be solved. We do so by obtaining prepotentials for (9) and using the flat values (10) for the D^a_u, D^a_p. We will therefore have to write down an action which leads to (11), which correspond to the linearity of A^l_α, $A_{\dot\alpha m}$ in the vectors $\boldsymbol{u}^{(l)}$ and $\boldsymbol{p}^{(m)}$, respectively.

The difficulty in implementing this programme is that the constraints (11) do not appear to be strong enough to ensure linearity of the associated A^l_α, $A_{\dot\alpha m}$ in $\boldsymbol{u}^l$ and $\boldsymbol{p}_m$, respectively. This is because quantities like $T^{[ij]}u_i^{(1)}u_j^{(2)}$ are invariant under the $SU(2)\times SU(2)$ notations and hence cannot be excluded from A^l_α or $A_{\dot\alpha m}$. We must enlarge the generators to include those in $SU(4)/SU(2)\times SU(2)$ in order to be able to deduce linearity. It is difficult to prevent including the whole set of generators of SU(4) in this way, so we will proceed by returning to the coset space approach. We note that this feature is also present in the light-line approach [11]. It is not a serious problem if we are not attempting to replace the totality of the set of constraints (1), but becomes so in the present formulation.

After we have chosen for H the various possibilities of the table, it seems that they all lead to effectively the same features (except CP^3, which cannot describe reality (2)), so we take as typical for H $SU(2)\times U(1)$, with analytic subspace comprising the variables $(x^{\alpha\dot\alpha}_A$, θ^{+a}_α, $\bar\theta^+_{\dot\alpha a}$ an appropriate subset of harmonic derivatives being $D^{++a}{}_b\equiv u_i^{+a}\partial/\partial u_i^{-b}-\bar u_i^{+a}\partial/\partial\bar u_i^{-b}$. Then the general solution of (9) at the linearised level is

$$A_\alpha = D_\alpha P + D^{3\ 4}_\alpha V^{-6} + D_\alpha(D^{3\gamma}V_\gamma - \bar D^3_{\dot\beta}V_{\dot\beta}) \tag{12}$$

with W of (9b) and (9c) now given the +2 U(1) charge,

$$W^{++} = D^4\bar D^4 V^{-6} \tag{13}$$

and $D^4\equiv\varepsilon^{\alpha\beta\gamma\delta}D_\alpha\wedge D_\beta\wedge D_\gamma\wedge D_\delta$, with $\varepsilon^{\alpha\beta\gamma\delta}$ being the completely antisymmetric tensor of $SL(2,C)\times SU(2)$, $D^{3\alpha}\equiv\varepsilon^{\alpha\beta\gamma\delta}D_{\beta\wedge}D_{\gamma\wedge}D_\delta$ and P is a real superfield. We expect that (12) can be covariantised by replacing D_α by $\mathscr{D}_\alpha$, $\bar D_{\dot\alpha}$ by $\bar{\mathscr{D}}_{\dot\alpha}$, and use of similar perturbation techniques as used for the $N=2$ superfield construction of $N=2$ SYM [8]; we propose to analyse this elsewhere.

In order to construct an action we must use the full superspace measure $(d^4x d^{16}\theta du)$, which has (mass) dimension 4 and U(1) charge 0. We will only consider the linearised solution (12) here, so we wish to obtain the linearised equation

$$D^{++a}_b W^{++} = 0 \tag{14}$$

as the equation of motion. That (14) is indeed the correct equation of motion follows from the fact that from (14) it follows that W^{++} can only be quadratic in u, and so A_α, $A_{\dot\alpha}$ only linear in them. Since V^{-6} has charge -6 and is of dimension 3 a quadratic term in V^- and W^{++} of the correct mass dimension cannot be constructed. It does not seem possible to construct a quadratic action involving V and suitable spinor derivatives which only allows linear terms in u in A_α. For example, the most natural expression for the action density is

$$(D^{++})^{2b}{}_d V^{-6}(D^{++})^{2a}{}_c D^{2cd}\bar{D}^2{}_{ab} V^{-6}. \tag{15}$$

This leads to field equations of the form

$$(D^{++})^{2(a}{}_{(b}(D^{++})^{2c)}{}_{d)} W^{++} = 0 \tag{16}$$

which are weaker than (14) in not excluding other than quadratic terms in W^{++}. The other allowed choices of H all give the same problem of constructing a suitable quadratic action.

In summary we have shown that the harmonic superspace approach has a number of possible choices of subgroup H in the construction of SU(4)/H. The analytic property is not compatible with that of reality, so that a prepotential formulation is needed, with actions obtained by integrating over the full superspace. Reality restricts H, and in the resulting allowed cases, although prepotentials were obtained, no action appears able to give the final set of constraints putting the theory on-shell.

We conclude that a constrained action may allow a viable way forward, with quantisation by the introduction of Lagrange multipliers to relax the constraints. This leads us closer to the constrained central charge formulation.

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